Title: Edge of Chaos Theory Explains the Mechanisms behind Smale Paradox in the Simplest Ever-Reported Bio-Inspired Memristor Oscillatory Network

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Abstract: As established by the second law of thermodynamics, an isolated system is unable to support complex phenomena. Conversely, a system, which communicates with the surrounding environment, may exhibit complex behaviors, provided some of its constitutive components are capable to amplify infinitesimal fluctuations in energy under suitable polarization [1], a property known as *Local Activity*. Back in 1974 the American luminary Stephen Smale [2] observed a counterintuitive phenomenon, later referred to as *Smale Paradox*, over the course of an experiment on a reaction-diffusion system.

Two identical 4th-order reaction cells, sitting on a common quiet state on their own, were found to undergo sustained limit-cycle oscillations when immersed in a coupling diffusive medium. An explanation for this unexpected phenomenon may only be found in the Theory of Local Activity [3], and recurring, particularly, to the *Edge of Chaos Theorem*, which asserts that a stable operating point Q of an isolated cell may be destabilized, as the cell is coupled to a dissipative environment, if and only if the isolated cell is capable to amplify a small-signal superimposed on Q, i.e. if and only if the isolated cell is both stable and locally-active, i.e. on the Edge of Chaos, at Q.

In this seminar we shall introduce the simplest ever-reported bio-inspired oscillatory network [4], consisting of two resistively-coupled and identical 2nd-order memristor cells, which supports the counterintuitive emergent phenomena, that mesmerized Stephen Smale in the seventies. The Smale paradox will be resolved here, once and for all, by demonstrating how static and dynamic patterns may develop in the reaction-diffusion system if and only if the isolated memristor oscillatory cell is biased on a stable and locally-active operating point.

An in-depth study, based upon linearization analysis and large-signal phase-portrait investigations, allows to draw a comprehensive picture for the local and global dynamics of the reaction cell, including a niobium oxide memristor [5], which features a peculiar S-shaped DC current-voltage locus, and is fabricated and characterized at NaMLab gGmbH [6]. This allows to develop a rigorous methodology to tune the design parameters of the two-cell array so as to induce diffusion-driven instabilities therein. This work sheds light on the precious role that nonlinear system-theory may assume in the years to come to support circuit designers in the exploration of the full potential of memristors in bio-inspired electronics.

References:

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