NOISE IN A SPECTRUM ANALYZER

by

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1. OBJECTIVE

The objective is to measure the noise figure of a spectrum analyzer with different methods and get familiar with noise and its statistical properties.

2. EQUIPMENT

You need spectrum analyzer, a 50 Ω termination, a signal generator, a coaxial cable. In the figures we refer to a specific brand (*hp*) of spectrum analyzer. However, any modern spectrum analyzer with digital display is adequate for the purpose of these experiments.

3. PROCEDURE

Switch on the spectrum analyzer and wait until warm up is complete (1 hour is sufficient). After warm up perform self-calibration. Then connect the 50 Ω termination to the input of the spectrum analyzer.
Use these settings, which are common to all the following experiments: Center frequency = 100 MHz, span = 0 Hz, input attenuation = 0 dB, resolution bandwidth (− 3 dB bandwidth, RBW) = 30 kHz, video bandwidth (VBW) = the largest available (e.g., 3 MHz).

3.1 Average level of the displayed noise in linear scale

Set linear scale and measurement unit microvolt (μV). Select the reference level so that the noise fully occupies the vertical range without exceeding the reference level (see Fig. 1). Set "sample" detector. Narrow the video bandwidth (or video average) so that noise appears as a smooth horizontal line. Record the average noise level $(\bar{m})_{dis}$ by using the display line and calculate the noise factor F through equation (4). Use the provisional value of the noise bandwidth as given by $B = 1{,}1 \cdot 30\,\text{kHz} = 33\,\text{kHz}$. A more accurate value of the noise bandwidth will be calculated at the last step, in §3.5. Finally, calculate the noise figure NF as $NF = 10\log_{10} F$. In the case represented in Fig. 1 we have $(\bar{m})_{dis} = 1{,}145\,\mu\text{V}$ and therefore, by using (4) we obtain $F = 252{,}8$ and $NF = 24{,}0\,\text{dB}$.

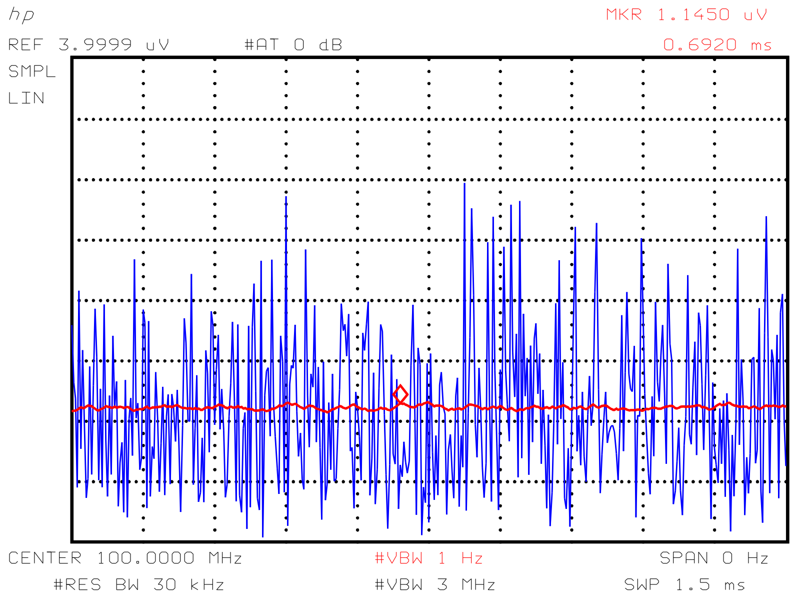

Fig. 1: Noise (blue trace, VBW = 3 MHz) and average noise level (red trace, VBW = 1 Hz) in linear scale.

3.2 Crest factor of the displayed noise

Use the same settings as in §3.1 and set the video bandwidth at its largest value. Take a single sweep and, by using the display line, determine the level L_{dis} that is exceeded by *n* noise peaks. To this purpose place the display line at the vertical position of the (*n* + 1)-th lower peak (see Fig. 2). The practical value of *n* is comprised between 5 and 10. Repeat the determination of L_{dis} at least ten times, by using the same value for *n*, and take the average value as the final result. Calculate the probability *p* of exceeding the level L_{dis} as $p = n/N$ where *N* is the total number of displayed values (*N* is also called "trace length", in the case represented in Fig. 2 we have $N = 401$ and $n = 7$ hence $p = 0{,}0175$). Calculate the crest factor C_F by using equation (9) and then the noise factor *F* by using equation (10) and the provisional value for *B*. Finally calculate the noise figure *NF*. See for example Fig. 2. We calculate $C_F = 2{,}012$ and $L_{dis} = 2{,}598\,\mu\text{V}$, therefore by using equation (10), we have $F = 252.5$ and $NF = 24{,}0\,\text{dB}$.

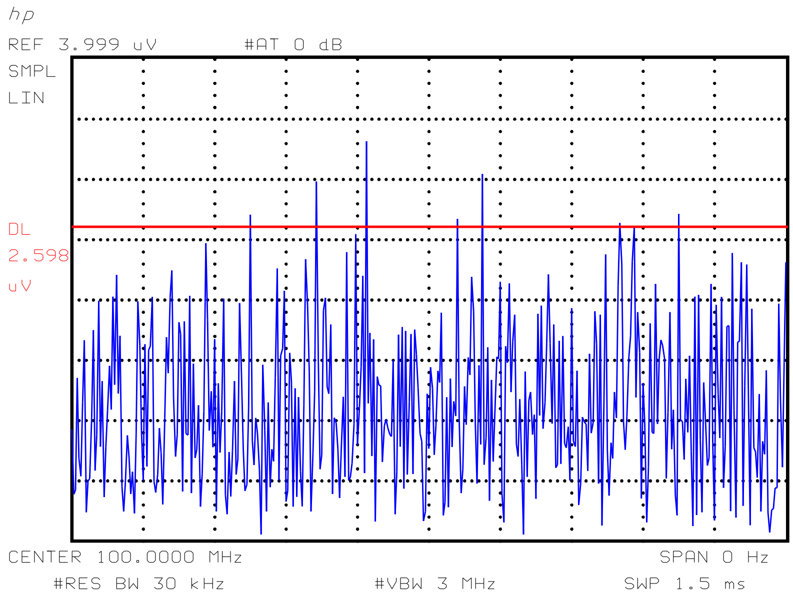

Fig. 2: Noise in linear scale (blue trace) and display line (red) at the eighth peak of noise.

3.3 Mean square value of the displayed noise

Use the same settings as in §3.2 . Verify that the video bandwidth is set to its largest value. Activate the marker and place it more or less at the center of the horizontal sweep (see Fig. 3). Run at least thirty single sweeps and record each marker value. Calculate the mean square value $\left(\overline{m^2}\right)_{dis}$ (sum of the squared values divided by the number of values). Then calculate the noise factor F by using equation (14) and the provisional value for B. Finally calculate the noise figure NF. In the case represented in Fig. 3 we obtained (thirty sweeps) $\left(\overline{m^2}\right)_{dis} = 1,730 \cdot 10^{-12}\,(\mu\mathrm{V})^2$ and therefore $F = 270,7$ and $NF = 24,3$ dB.

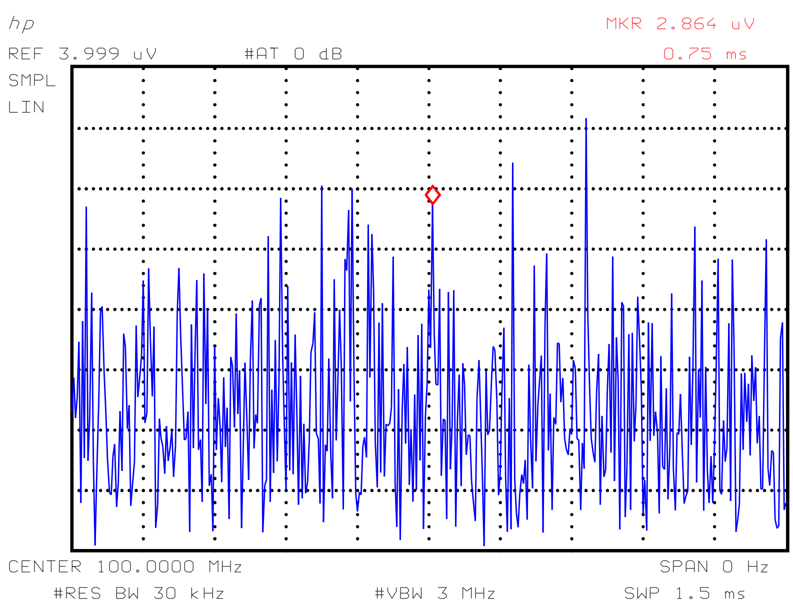

Fig. 3: Noise in linear scale. Marker captures one random value of noise.

3.4 Average level of the displayed noise in logarithmic scale

Set logarithmic scale in dBm units and 10 dB per division. Set the reference level so that the noise is mostly over the bottom division of the vertical scale. Set “sample” detector. Narrow the video bandwidth (or video average) in order to smooth the noise, as shown in Fig. 4, and easily identify the average noise level by using the marker or the display line, see §3.1. Record the average noise level as *ANL* and calculate the noise figure *NF* by using equation (21) and the provisional value of the noise bandwidth *B*. If, as in Fig. 4, $ANL = -107{,}07\,\text{dBm}$ then $NF = 24{,}2\,\text{dB}$.

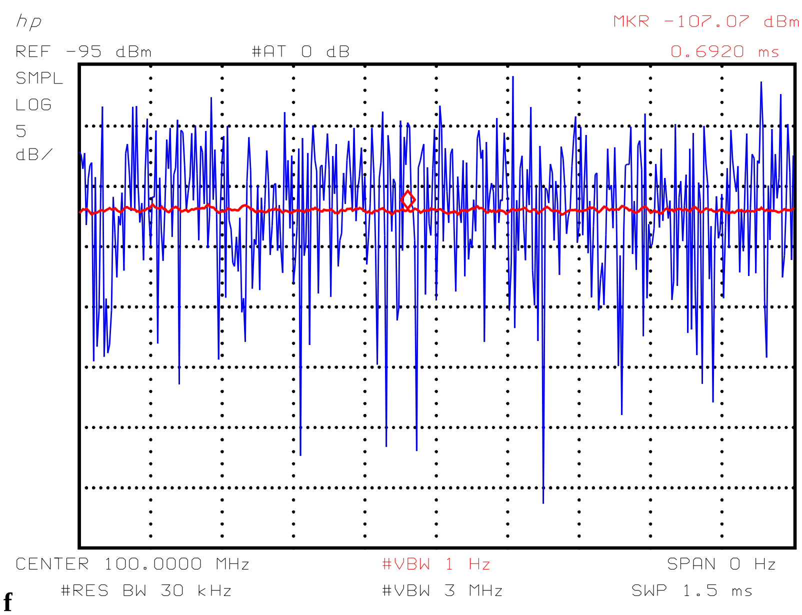

Fig. 4: Noise in logarithmic scale (blue trace, VBW = 3 MHz) and *ANL* (red trace, VBW = 1 Hz).

3.5 Equivalent noise bandwidth

Set linear scale and measurement unit millivolt (mV). Set “peak” detector. Set 10 dB internal attenuation. Apply a 5 MHz sinusoidal signal at the input of the spectrum analyzer. The amplitude of the sinusoidal signal is 10 mV peak to peak. Verify that the resolution bandwidth is approximately 30 kHz (for example by using the “marker delta” or similar function). Set the center frequency so that the peak of the signal is at the center of the horizontal sweep. Set the reference level so that the peak of the signal touches the reference level (see Fig. 5). Set the span so that the tails of the bell-shaped response of the intermediate-frequency filter is undistinguishable from zero in the leftmost and rightmost one and half division. Narrow the video bandwidth in order to smooth the trace (if some noise is visible in the tails of the response of the filter). Record at least twenty equally frequency-spaced values H_i of the filter response and the corresponding frequencies.

Record the peak value of the response of the filter H_0 and divide each recorded value by H_0, thus obtaining $h_i = H_i / H_0$.

Calculate the square of each normalized value h_i^2 and then the quantity (Δf is the frequency spacing):

$$I = \left(\frac{h_1^2}{2} + h_2^2 + h_3^2 + \ldots + \frac{h_m^2}{2} \right) \Delta f \ .$$

The result *I* is an approximation to the noise bandwidth *B* (see §4.5) which can be used in place of the 33 kHz provisional value. In the case represented in Fig. 5 it results $B = 33{,}1\,\text{kHz}$, which is quite close to the provisional 33 kHz value. Integration over the full trace length (401 values) affects only the least significant figure (33,2 in place of 33,1 kHz).

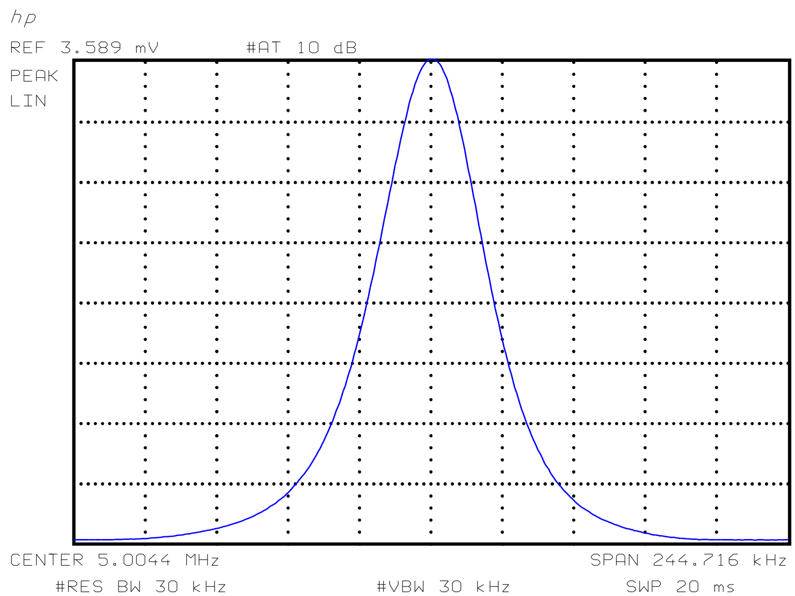

Fig. 5: Frequency response (magnitude) of the IF filter. The nominal RBW is 30 kHz, the measured value is 29,0 kHz. The measured noise bandwidth (integration over 20 values) is 33,1 kHz.

4. THEORY

The noise detected by a spectrum analyzer has a probability density function (pdf) given by

$$f(m) = \begin{cases} \dfrac{m}{v_n^2} \cdot \exp\left(-\dfrac{m^2}{2v_n^2}\right) & \text{if } m \geq 0 \\ 0 & \text{if } m < 0 \end{cases}, \tag{1}$$

where m is the instantaneous value of the noise, $v_n^2 = FkTBR$, F is the noise factor of the spectrum analyzer, k is the Boltzmann constant ($k = 1{,}38 \cdot 10^{-23}$ J/K), T is the absolute temperature (conventionally $T = T_0 = 290\,\mathrm{K}$), B is the equivalent noise bandwidth of the spectrum analyzer and $R = 50\,\Omega$. $v_n = \sqrt{FkTBR}$ is the equivalent noise root-mean-square (rms) voltage applied across the input terminals of the spectrum analyzer. The equivalent noise power at the input of the spectrum analyzer is $P_n = FkTB = v_n^2 / R$.

The internal attenuation of the spectrum analyzer is assumed to be 0 dB (i.e., $A = 1$, where A is the power attenuation in linear scale). If the internal attenuation is increased then also the noise factor increases (by the factor A, i.e. the noise factor becomes $A \cdot F$). The noise factor of the spectrum analyzer is usually (and also here) evaluated by setting 0 dB internal attenuation.

4.1 Average level of the displayed noise in linear scale

The average noise level in linear scale is obtained as

$$\bar{m} = \int_0^\infty m \cdot f(m)\,dm = \sqrt{\frac{\pi}{2}} \cdot v_n, \tag{2}$$

where $f(m)$ is given by (1). Since the displayed level is the input level divided by $\sqrt{2}$ (the spectrum analyzer is calibrated to display to rms value of an equivalent sinusoidal input signal), then the displayed average noise level is

$$(\bar{m})_{dis} = \frac{\bar{m}}{\sqrt{2}} = \frac{\sqrt{\pi}}{2} \cdot v_n, \tag{3}$$

and

$$F = \frac{4}{\pi kTBR} \cdot \left(\overline{m}\right)_{dis}^{2} . \tag{4}$$

4.2 Crest factor of the displayed noise

The probability p that the noise detected by the spectrum analyzer exceeds the level L is given by

$$p = \int_{L}^{\infty} f\left(m\right) dm . \tag{5}$$

Substituting (1) into (5) and integrating we have

$$p = \exp\left(-\frac{L^2}{2v_n^2}\right) \tag{6}$$

and then

$$v_n^2 \cdot \ln\frac{1}{p} = \frac{L^2}{2} . \tag{7}$$

Since $L = \sqrt{2} L_{dis}$ where L_{dis} is the displayed level that is exceeded with probability p, then

$$v_n^2 \cdot \ln\frac{1}{p} = L_{dis}^2 . \tag{8}$$

We can define the crest factor C_F of the displayed noise as

$$C_F = \frac{L_{dis}}{v_n} = \sqrt{\ln\left(1/p\right)} . \tag{9}$$

and

$$F = \frac{1}{kTBR} \cdot \frac{L_{dis}^2}{C_F^2} . \tag{10}$$

4.3 Mean square value of the displayed noise

The mean square value of the detected noise is given by

$$\overline{m^2} = \int_{0}^{\infty} m^2 f\left(m\right) dm . \tag{11}$$

Substituting (1) into (11) we obtain, solving the integral by parts,

$$\overline{m^2} = 2v_n^2 . \tag{12}$$

Since the displayed levels are in terms of the rms value of the equivalent sinusoidal signal then

$$\left(\overline{m^2}\right)_{dis} = v_n^2 \tag{13}$$

and

$$F = \frac{\left(\overline{m^2}\right)_{dis}}{kTBR} . \tag{14}$$

4.4 Average level of the displayed noise in logarithmic scale

The average noise level in logarithmic scale is obtained as

$$\overline{20\log_{10}\left(m\right)} = \int_{0}^{\infty} 20\log_{10}\left(m\right) \cdot f\left(m\right) dm . \tag{15}$$

Substituting (1) into (15) we obtain

$$\overline{20\log_{10}\left(m\right)} = \int_{0}^{\infty} 20\log_{10}\left(m\right) \cdot \frac{m}{v_n^2} \cdot \exp\left(-\frac{m^2}{2v_n^2}\right) dm . \tag{16}$$

If the variable of integration *m* is replaced by $x = m/v_n$ then we have

$$\overline{20\log_{10}(m)} = \int_0^\infty 20\log_{10}(x)\cdot x\cdot \exp\left(-\frac{x^2}{2}\right)dx + 20\log_{10}(v_n), \tag{17}$$

where we used the fact that $\int_0^\infty f(m)dm = 1$. Numerically computing the integral at the right side of (17) we obtain

$$\overline{20\log_{10}(m)} = 0{,}50 + 20\log_{10}(v_n) \tag{18}$$

and, taking to account that the displayed level is expressed in terms of the rms value of an equivalent sinusoidal input, we have

$$\left(\overline{20\log_{10}(m)}\right)_{dys} = \overline{20\log_{10}(m)} - 3{,}01 = -2{,}51 + 20\log_{10}(v_n). \tag{19}$$

The average noise level in logarithmic scale is usually referred to with the acronym ANL and is usually expressed in dBm, therefore from (19) we obtain

$$ANL = -2{,}51 + (FkTB)_{\mathrm{dBm}}. \tag{20}$$

The noise figure of the receiver *NF* is obtained from (20) as

$$NF = ANL + 2{,}51 - (kTB)_{\mathrm{dBm}}. \tag{21}$$

4.5 Equivalent noise bandwidth

The noise power P_n delivered to a 50 Ω load *R*, connected at the output of a two-port network, whose power gain is frequency dependent and given by $G(f)$, is

$$P_n = \int_0^\infty S(f)G(f)df \tag{22}$$

where $S(f)$ is the noise spectral density (expressed in W/Hz) delivered to the 50 Ω input of the network. If the input noise is white then it is constant with frequency and $S(f) = S$, therefore

$$P_n = S\int_0^\infty G(f)df \tag{23}$$

Further, if the frequency response of the network is bell-shaped and the maximum value of the gain over frequency is G_0 then

$$P_n = SG_0B \tag{24}$$

where

$$B = \frac{1}{G_0}\int_0^\infty G(f)df \tag{25}$$

and *B* is the equivalent noise bandwidth of the network. The equivalent noise bandwidth can be also expressed in terms of the magnitude of the voltage transfer function of the network $H(f)$. Indeed, since $G(f) = |H(f)|^2$ then, from (25) we obtain

$$B = \frac{1}{H_0^2}\int_0^\infty |H(f)|^2\, df, \tag{26}$$

where H_0 is the maximum value of $|H(f)|$.

5. CONCLUSION

Four different methods are here described to measure the noise figure of a spectrum analyzer. In order to obtain compatible results among the methods the following recommendations should be implemented. Noise is a random process therefore averaging over a greater sample size leads to more precise results. Take this into account especially for the methods described in §3.2 and in §3.3. For example, the relative precision of the estimate of the noise factor achievable by averaging thirty values is $1/\sqrt{30}$, i.e. about 18 % (or 0,7 dB in terms of noise figure). Further, remember that a spectrum analyzer is a physical device, not a random number generator based on a mathematical algorithm. As a consequence switching instrument settings (center frequency, resolution bandwidth, attenuation, reference level …) when passing from one method to the other introduces measurement inaccuracies. This is particularly true when switching the scale from linear to logarithmic, as required in order to pass from the method described in §3.3 to that described in §3.4. In conclusion the agreement among the four noise figure values depends on both the care with which the measurements are performed and on the quality of the spectrum analyzer. If experiments are performed with the due care and a modern spectrum analyzer is used then a disagreement of up to 1 dB is absolutely acceptable, while more than 2 dB is questionable.

6. REFERENCES

[1] Keysight Technologies, Application Note 150, “Spectrum Analysis Basics,” 2015.
[2] Agilent Technologies, Application Note 1303, “Spectrum Analyzer Measurements and Noise,” 2003.