A Double Inequality for the Equivalent Impulse Bandwidth

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***Abstract*—In this paper, we derive a double inequality that permits one to obtain a lower and an upper limit for the value of the impulse bandwidth of a measuring receiver. The limits are the reciprocal of the integral of the relative envelope of the impulse response (lower limit) and the integral of the relative frequency response (upper limit) of the intermediate frequency (IF) filter. Since the limits are relative quantities, their evaluation does not require the use of a calibrated generator, the only significant sources of error being receiver's vertical scale nonlinearity and noise proximity. Here, the deviation between the impulse bandwidth and its limits is quantified for practical IF filter configurations. The dominant contributions to measurement uncertainty are identified and suggestions for reducing their magnitude are also offered.**

***Index Terms*—Impulse bandwidth, pulse measurements, radar measurements, receivers, uncertainty.**

I. Introduction

The RESPONSE of receivers to impulses has been the object of investigation both in the past and in recent times. Several technical papers [1]–[5], application notes [6], [7], and standards [8]–[10] deal with the subject making use of the concept of *impulse bandwidth* and introducing different impulse definitions and impulse bandwidth measurement techniques. The need to quantify the electromagnetic interference produced by ultrawideband devices has led to a renewed interest in the analysis of receiver response to impulses [11]. Further, increasing attention to the issue of measurement uncertainty has stimulated the reconsideration of well-established measurement techniques in order to identify, quantify, and possibly, minimize the dominant contributions to uncertainty. Impulse measurements are no exception in this respect.

The impulse bandwidth B_{imp} of a receiver is defined as follows [9], [10]:

$$B_{\mathrm{imp}} = \frac{A(t)|_{\max}}{2H_0 \mathrm{IS}} \tag{1}$$

where $A(t)|_{\max}$ is the peak value of the envelope $A(t)$ of the response of the intermediate frequency (IF) filter to an input impulse $v_i(t)$ having area (or impulse strength) IS, i.e., $\mathrm{IS} = \int_0^\infty v_i(t)dt$, and H_0 is the magnitude of the frequency response of the IF filter at its center frequency. Among the various methods adopted to evaluate the impulse bandwidth, two are of specific interest here, namely: 1) the integration of the normalized envelope of the impulse response of the IF filter and 2) the integration of the normalized frequency response of the IF filter, both responses in linear scale. The first method is described in [8] and [10]. It is interesting to observe that this method was taken as the operative definition of impulse bandwidth in place of (1) in [8] and in the subsequent documents [2] and [3]. The second method (integration of the frequency response) is based on an upper bound on the impulse response of a real and causal system, which was obtained in [12] [(10) with $W_A(t) = 0$ and $R_A(\omega) = 0$], [13] [(3) with $u_1(t) = 0$ and $H_1(\omega) = 0$], and [14] [Theorem 2, in the special case where $W_1(t) = 0$ and $T_e(\omega) = T(\omega)$]. The same result was also derived in [1], where it was pointed out that the bound provides an upper limit to impulse bandwidth, which can be closely approximated for specific IF filter designs. The method is also mentioned in [3] and is adopted by standards and technical documents, such as [7] and [9], to obtain an approximation to the equivalent impulse bandwidth.

The scope of this paper is twofold. First, in Section III, we derive a lower and an upper limit to the equivalent impulse bandwidth along with the requirements, for the IF filter response, that must be met in order that the impulse bandwidth may coincide with one or the other limit or both. Second, in Section IV, the limits are quantified for commonly adopted IF filter designs. A discussion is also included in that section concerning the measurement uncertainty inherent to the experimental determination of the limits. The basic equations necessary for the derivation of the results in Section III are introduced in Section II.

II. Envelope Detection of Bandpass Signals

We here introduce the properties of the envelope of bandpass signals relevant to the analysis developed in the following section. These properties are general, in that they apply to all real and causal bandpass systems.

Let us consider a single impulse $v_i(t)$ applied at the input of the receiver. It is assumed here that the superheterodyne frequency conversion of the input signal is equivalent to the rigid transfer of the frequency response of the IF filter (briefly "the filter," from now onward) from the IF to the tuning frequency of the receiver ω_S. Thus, ω_S also corresponds to the center frequency of the filter. If $H(\omega)$ is the complex transfer function of the filter, then $H_0 = |H(\omega_S)|$. Also, if $h(t)$ is the filter response to the Dirac's impulse of unit area $\delta(t)$, then, we have $h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)e^{j\omega t}d\omega$.

Manuscript received May 21, 2009; revised October 19, 2009; accepted January 12, 2010. Date of publication February 25, 2010; date of current version August 18, 2010 .

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Digital Object Identifier 10.1109/TEMC.2010.2041237

The impulse response of a bandpass system [15, Th. 3, p. 123] is as follows:

$$h(t) = \mathrm{Re}\{\hat{h}(t)e^{j(\omega_S t+\varphi_S)}\}$$

where $\varphi_S = \arg[H(\omega_S)]$. The time-domain function $\hat{h}(t)$ is termed as the *complex envelope* of the impulse response. If the magnitude of the response $|H(\omega)|$ is negligibly small outside the interval $|\omega - \omega_S| < \Omega/2$, then

$$\hat{h}(t) = \frac{1}{2\pi}\int_{-\Omega/2}^{\Omega/2} \hat{H}(\omega)e^{j\omega t}d\omega \tag{2}$$

where $\hat{H}(\omega) = Z_h(\omega+\omega_S)e^{-j\varphi_S}$, $Z_h(\omega) = 2H(\omega)U(\omega)$, and $U(\omega)$ represents the Heaviside step function. $\hat{H}(\omega)$ is said to be the *equivalent low-pass filter* of the bandpass $H(\omega)$. Note that

$$\hat{H}(0) = 2H(\omega_S)e^{-j\varphi_S}. \tag{3}$$

In the case where $H(\omega)$ is a *symmetric bandpass filter*, i.e., $|H(\omega)|$ is even and $\arg[H(\omega)] - \varphi_S$ is odd around ω_S, then $\hat{H}(-\omega) = \hat{H}^*(\omega)$ and $\hat{h}(t)$ is real.

The filter output signal $v_o(t)$ is given by [15, Th. 4, p. 123]

$$v_o(t) = \mathrm{Re}\{\hat{v}_e(t)e^{j(\omega_S t+\varphi_S)}\}$$

where $\hat{v}_e(t)$ is the *complex envelope* of the output voltage and

$$\hat{v}_e(t) = \int_0^\infty \hat{h}(t-\theta)v_i(\theta)e^{-j\omega_S\theta}d\theta. \tag{4}$$

The *envelope* of the output voltage is, by definition, $A(t) = |\hat{v}_e(t)|$. Equations (2) and (4) form the basis of the analysis in the next section.

III. Derivation of a Double Inequality for the Impulse Bandwidth

We here derive an upper and a lower limit for the impulse bandwidth of a receiver. As an intermediate step, it is necessary to evaluate the peak value of the envelope of the receiver's response to a wideband input signal. A wideband signal is intended as a signal, whose spectral density is constant through the bandwidth of the receiver or, equivalently, whose duration is short with respect to the duration of the envelope of the impulse response of the receiver. In our derivations $v_i(t)$ is a Dirac's delta impulse of area IS. This greatly simplifies the analysis while keeping it informative. Indeed, the results obtained are also valid in the case of a finite (nonzero) duration and finite-amplitude input impulse provided that $|H(\omega)|$ is symmetric around ω_S, as is shown in the Appendix.

Let us consider (4). If $v_i(t) = \mathrm{IS}\,\delta(t)$, then $\hat{v}_e(t) = \mathrm{IS}\,\hat{h}(t)$. Substituting into (2) and taking the magnitude, we obtain

$$A(t) = \mathrm{IS}\frac{1}{2\pi}\left|\int_{-\Omega/2}^{\Omega/2}\hat{H}(\omega)e^{j\omega t}d\omega\right|. \tag{5}$$

Hence, $A(t)|_{\max} \le \mathrm{IS}[1/2\pi\int_{-\Omega/2}^{\Omega/2}|\hat{H}(\omega)|d\omega]$. By using the bandpass assumption, we have

$$A(t)|_{\max} \le 2\mathrm{IS}\left[\frac{1}{2\pi}\int_0^\infty |H(\omega)|d\omega\right]. \tag{6}$$

We have just obtained an *upper limit* on the maximum value of the envelope of the filter output. We observe that if the phase response of the filter is linear, at least within the frequency range, where $|\omega - \omega_S| < \Omega/2$, we have $\arg[\hat{H}(\omega)] = -\tau_g\omega$, where τ_g is the *group delay*. Hence, the integral in (5) can be rewritten as follows:

$$\left|\int_{-\Omega/2}^{\Omega/2}\hat{H}(\omega)e^{j\omega t}d\omega\right| = \left|\int_{-\Omega/2}^{\Omega/2}|\hat{H}(\omega)|e^{j\omega(t-\tau_g)}d\omega\right|$$

from which we conclude that

$$A(t)|_{\max} = A(\tau_g) = 2\mathrm{IS}\left[\frac{1}{2\pi}\int_0^\infty |H(\omega)|\,d\omega\right].$$

Thus, if the phase response of the filter is linear, the maximum amplitude of the envelope is reached at $t = \tau_g$ and its value corresponds to the upper limit in (6).

By integration of the complex envelope of the output voltage, we obtain

$$\int_0^\infty \hat{v}_e(t)dt = \mathrm{IS}\int_0^\infty \hat{h}(t)dt. \tag{7}$$

Since $\hat{H}(0) = \int_0^\infty \hat{h}(t)dt$, from (7), we have $\int_0^\infty \hat{v}_e(t)dt = \mathrm{IS}\hat{H}(0)$. By using (3), we obtain

$$\int_0^\infty \hat{v}_e(t)dt = 2\mathrm{IS}H(\omega_S)e^{-j\varphi_S}. \tag{8}$$

Taking the magnitude of the left and right side of (8), we find

$$\int_0^\infty A(t)dt \ge 2\mathrm{IS}H_0 \tag{9}$$

where we used $|\int_0^\infty \hat{v}_e(t)dt| \le \int_0^\infty |\hat{v}_e(t)|dt = \int_0^\infty A(t)dt$. Note that if $\hat{v}_e(t)$ is real and positive, the identity applies in (9). $\hat{v}_e(t)$ is real for a symmetric, bandpass filter. Further, $\hat{v}_e(t)$ is positive if, for example, $H(\omega)$ corresponds to a single or a cascade of tuned stages (see Section IV-B). Now, substituting (6) and (9) into (1), we finally obtain

$$\frac{A(t)|_{\max}}{\int_0^\infty A(t)dt} \le B_{\mathrm{imp}} \le \frac{1/2\pi\int_0^\infty |H(\omega)|d\omega}{H_0}. \tag{10}$$

IV. Evaluation of the Limits in Practical Cases

It is of interest to evaluate the impulse bandwidth and its limits, as defined by the double inequality (10), in the more common cases of practical IF filter designs. Two IF filter configurations will be considered: the cascade of two ideal, critically coupled, tuned circuits, and the cascade of n single-tuned circuits. These or similar designs are implemented in receiver architectures and their ideal behavior is frequently adopted as a reference (see [1], [2], [7], and [9]) for the prediction of the response of receivers to standard signals (i.e., sine wave, impulse, and noise).

A. Cascade of Two Ideal, Critically Coupled, Tuned Circuits

In this case, the equivalent low-pass filter is

$$\hat{H}(\omega) = \frac{8\omega_0^4}{[(\omega_0+j\omega)^2+\omega_0^2]^2} \tag{11}$$

and the corresponding complex envelope is

$$\hat{h}(t) = 4\omega_0[e^{-\omega_0 t}(\sin\omega_0 t - \omega_0 t\cos\omega_0 t)] \tag{12}$$

where $\omega_0 = \pi B_6/\sqrt{2}$ and B_6 is the 6 dB bandwidth of the filter. To simplify the notation, $H_0 = 1$ was assumed, and then, $\hat{H}(0) = 2$. Since the filter is symmetric, $\hat{h}(t)$ is real. Also note that $\hat{h}(t)$ has an oscillating waveform. The envelope of the impulse response is as follows:

$$A(t) = 4\text{IS}\omega_0 e^{-\omega_0 t}|\sin\omega_0 t - \omega_0 t\cos\omega_0 t|. \tag{13}$$

From numerical computation, we have that

$$A(t)|_{\max} = 1.048(2\text{IS})B_6$$

then from (1) $B_{\text{imp}} = 1.048B_6$. Also, since

$$\int_0^\infty A(t)dt = 1.133(2\text{IS})$$

then the lower limit on B_{imp} [see (10)] is $0.883B_{\text{imp}}$. Further, since

$$\frac{1}{2\pi}\int_0^\infty |H(\omega)|d\omega = \frac{\pi B_6}{2\sqrt{2}} = 1.111B_6$$

then the upper limit is $1.060B_{\text{imp}}$.

It is interesting to observe that from (11), we have

$$\tau_g = \frac{2}{\omega_0} = \frac{2\sqrt{2}}{\pi B_6}$$

and from (13), we have

$$A(\tau_g) = 1.047(2\text{IS})B_6.$$

Hence, $A(\tau_g)$ provides a very good approximation of $A(t)|_{\max}$ (relative deviation less than 0.1%).

B. Cascade of n-Identical Single-Tuned Circuits

The equivalent low-pass filter of this configuration is

$$\hat{H}(\omega) = \frac{2\omega_0^n}{(\omega_0 + j\omega)^n} \tag{14}$$

and the complex envelope is

$$\hat{h}(t) = 2\omega_0\frac{(\omega_0 t)^{n-1}e^{-\omega_0 t}}{(n-1)!} \tag{15}$$

where $\omega_0 = \pi B_6/\sqrt{4^{1/n} - 1}$. Again $H_0 = 1$, then $\hat{H}(0) = 2$. Note that $\hat{h}(t)$ is a unidirectional impulse, hence we have $A(t) = v_e(t)$ and the lower limit of (10) corresponds to B_{imp}. Since $A(t)$ reaches its maximum value at the time instant $(n-1)/\omega_0$, from (15)

$$A(t)|_{\max} = 2\text{IS}\frac{\pi(n-1)^{n-1}e^{-(n-1)}}{\sqrt{4^{1/n}-1}(n-1)!}B_6$$

and from (1)

$$B_{\text{imp}} = \frac{\pi(n-1)^{n-1}e^{-(n-1)}}{\sqrt{4^{1/n}-1}(n-1)!}B_6. \tag{16}$$

If n increases, then B_{imp} decreases, tending to

$$1/2\cdot\sqrt{\pi/\ln 2}B_6 = 1.064B_6$$

TABLE I
IMPULSE BANDWIDTH AND THE CORRESPONDING LOWER AND UPPER LIMITS, AS GIVEN BY (10), FOR THE CASCADE OF n-IDENTICAL TUNED FILTER CIRCUITS

n	$\frac{B_{imp}}{B_6}$	$\frac{\text{Lower bound}}{B_{imp}}$	$\frac{\text{Upper bound}}{B_{imp}}$
1	1.814	1	∞
2	1.156	1	1.359
3	1.109	1	1.176
4	1.094	1	1.116
5	1.086	1	1.086
6	1.081	1	1.069
7	1.078	1	1.057
8	1.076	1	1.049
9	1.075	1	1.042
10	1.073	1	1.038

as n tends to infinity. In Table I, the values of B_{imp} are reported together with the lower and upper limits of (10) for the values of n comprised between 1 and 10.

Common filter configurations are obtained cascading four or five stages [7]. The values of B_{imp} and their limits, corresponding to a number of stages other than four and five, are however instructive. For example, we see that the upper limit is infinity for a single-tuned stage. This is due to the magnitude of the filter response, being asymptotically proportional to $1/\omega$, as ω tends to infinity. Also, we observe that the upper limit slowly decreases for values of n greater than four, but it still produces an overestimate of about 5% when $n = 8$.

The group delay is $\tau_g = n/\omega_0 = n\sqrt{4^{1/n} - 1}/\pi B_6$ and the relative deviation between $A(\tau_g)$ and $A(t)|_{\max}$ is 13% when $n = 4$, 10% for $n = 5$ and reduces to 5% when $n = 10$.

C. Summary and Comments

We summarize here the results derived in this section and we offer some comments concerning measurement uncertainty. In the case where the filter is realized through cascading two critically coupled tuned circuits, the limits for B_{imp}, as obtained from (10), are -11.7% and $+6.0\%$. Then, if one measures both limits and calculates the mean value, this deviates from B_{imp} by less than $+3\%$. If the filter is obtained by cascading simple tuned stages, then the complex envelope of the impulse response is (real and) unidirectional and the value of B_{imp} corresponds to the lower limit in (10). According to our experience, this is the most frequent case (unidirectional envelope of the impulse response), therefore the estimate corresponding to the lower limit usually represents the preferred option.

The main contributions to the measurement uncertainty of the lower and upper limits originate both from the deviation from linearity of the vertical scale of the receiver and from the presence of noise. The effect of the scale nonlinearity is mitigated in part by the fact that the limits are obtained through integration, and then, errors cancel out each other (here zero-mean linearity error is assumed). However, the presence of noise manifests itself through a pedestal value, which adds up to the integrand, thus enlarging the interval defined by (10), i.e., the measured lower limit is smaller and the measured upper limit is greater than they should be in absence of noise. In order to minimize the effect of noise, the average noise level N should be separately measured when input signal is absent, and then, subtracted from the average[1] displayed signal S_{dys} (the displayed envelope of the filter pulse response or frequency response), according to the following formula:

$$S = \begin{cases} \sqrt{S_{dys}^2 - N^2}, & \text{if } S_{dys} \geq N \\ 0, & \text{if } S_{dys} < N \end{cases} \tag{17}$$

where S is the quantity to be integrated to obtain the lower and upper limits. Equation (17) represents an approximation (nearly 4% maximum error) of the first-order moment of the Rice probability density function [16].

V. Conclusion

The availability of different options to measure impulse bandwidth is very useful because the span of technical applications is so wide that impulse bandwidth values may range from hundreds of hertz to tens of megahertz, depending on the specific purpose. Methods offering straightforward implementation and low measurement uncertainty are to be preferred.

The impulse bandwidth of a receiver, as defined in (1), is demonstrated to be bounded between the theoretical limits given by the double inequality (10), which is very general. In the case of practical IF filter designs, the deviation between the limits and the definition is very small (from zero to few percents). The mathematical expression of the limits describes measurement methods that can be readily implemented in order to derive an estimate of the impulse bandwidth. The methods are intrinsically immune from absolute and random errors, being based on the integration of a relative quantity. A formula to correct for the systematic additive error due to the intrinsic noise of the receiver is provided. Further, the evaluation of the integrals is greatly facilitated and made very accurate by the availability in modern receivers of the numerical values of the displayed trace and the same applies to the time-domain signal extracted from the video output of the receiver and sent to a digital oscilloscope.

Measurement campaigns and interlaboratory comparisons would be highly welcome with the scope to assess the limits of accuracy of the determination of impulse bandwidth, using different measurement methods and covering the wide range of bandwidth values involved in modern applications.

Appendix

We consider (4) and allow for a smooth variation of $\hat{h}(t)$ during the duration of $v_i(t)$. We assume that in this time interval, $\hat{h}(t-\theta)$ can be well approximated by its first-order Taylor expansion around $\theta = 0$, i.e., $\hat{h}(t-\theta) = \hat{h}(t) - \hat{h}'(t)\theta$. Substituting into (4), we obtain

$$\hat{v}_e(t) = \hat{h}(t)\int_0^\infty v_i(\theta)e^{-j\omega_S\theta}d\theta - \hat{h}'(t)\int_0^\infty \theta\, v_i(\theta)e^{-j\omega_S\theta}d\theta. \tag{A1}$$

[1] Averaging is obtained through multiple acquisitions of the displayed signal. The video over resolution bandwidth ratio shall be large (e.g., ten or more) in order to avoid distortion due to video filtering, particularly when the impulse is applied at the input of the receiver.

Now, since

$$V_i(\omega_S) = \frac{1}{2\pi}\int_{-\infty}^{\infty} v_i(t)e^{-j\omega_S t}dt$$

$$\hat{h}'(t) = \frac{j}{2\pi}\int_{-\Omega/2}^{\Omega/2} \omega\hat{H}(\omega)e^{j\omega t}d\omega$$

and

$$\int_0^\infty \theta\, v_i(\theta)e^{-j\omega_S\theta}d\theta = jV_i'(\omega_S)$$

we obtain from (A1)

$$\hat{v}_e(t) = V_i(\omega_S)\hat{h}(t) + V_i'(\omega_S)\int_{-\Omega/2}^{\Omega/2}\omega\hat{H}(\omega)e^{j\omega t}d\omega. \tag{A2}$$

Note that the term $V_i(\omega_S)\ \hat{h}(t)$ in (A2) corresponds to IS$\hat{h}(t)$ in (11). Hence, if the correction term

$$V_i'(\omega_S)\int_{-\Omega/2}^{\Omega/2}\omega\hat{H}(\omega)e^{j\omega t}d\omega$$

in (A2) were negligible, all the results obtained in the main text assuming an ideal Dirac's impulse at the filter input would be valid also for a real wideband input impulse, once taken $V_i(\omega_S)$ in place of IS. To show this, we evaluate the magnitude of the correction term at $t = \tau_g$, i.e., at the instant where the envelope approximately reaches its maximum value. We have

$$|\hat{v}_e(\tau_g) - V_i(\omega_S)\hat{h}(\tau_g)| \approx \left|V_i'(\omega_S)\int_{-\Omega/2}^{\Omega/2}\omega|\hat{H}(\omega)|d\omega\right| \tag{A3}$$

where we used $\hat{H}(\omega) \approx |\hat{H}(\omega)|e^{-j\omega\tau_g}$. Now, since most practical filter configurations have symmetric frequency response, $\int_{-\Omega/2}^{\Omega/2}\omega|\hat{H}(\omega)|d\omega = 0$ and the desired conclusion follows.

Acknowledgment

The authors would like to thank the reviewers for their constructive criticism and suggestions.

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